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QUANTUM-TO-CLASSICAL TRANSITION FOR FLUCTUATIONS IN THE EARLY UNIVERSE

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Abstract

According to the inflationary scenario for the very early Universe, all inhomogeneities in the Universe are of genuine quantum origin. On the other hand, looking at these inhomogeneities and measuring them, clearly no specific quantum mechanical properties are observed. We show how the transition from their inherent quantum gravitational nature to classical behaviour comes about – a transition whereby none of the successful quantitative predictions of the inflationary scenario for the present-day universe is changed. This is made possible by two properties. First, the quantum state for the spacetime metric perturbations produced by quantum gravitational effects in the early Universe becomes very special (highly squeezed) as a result of the expansion of the Universe (as long as the wavelength of the perturbations exceeds the Hubble radius). Second, decoherence through the environment distinguishes the field amplitude basis as being the pointer basis. This renders the perturbations presently indistinguishable from stochastic classical inhomogeneities.

Several years ago, one could not even think about the idea that a thorough understanding of structure formation in the Universe would necessarily entail a discussion of fundamental questions in quantum theory. And yet, this is what currently happens.

How does this come about? Scenarios which include an accelerated expansion of the Universe at an early stage (an “inflationary phase”) provide a natural, quantitative, mechanism for structure formation [1]. The origin of this structure can be traced back to unavoidable *quantum* fluctuations of some scalar field ϕ . These fluctuations then lead – together with analogous scalar fluctuations in the metric – to anisotropies in the cosmic background radiation. In addition, there are relict gravitational waves originating from tensor fluctuations in the metric. The COBE-mission and future projects such as the Planck Surveyor satellite mission are able to observe these anisotropies and possibly test the above scenario [1, 2].

Usually, classical properties for a certain system emerge by interaction of this system with its natural environment. This process is called decoherence (see [3] for a comprehensive review). Its main characteristics are its ubiquity and its irreversibility. One would expect that this process is also of utmost importance for the fluctuations in the early Universe, and in fact detailed investigations were performed in this respect (see [4] and the references therein).

On the other hand, the case was made that as a result of the *special* time evolution of the system – leading to a highly squeezed quantum state – no environment is needed in order to make quantitative predictions, since no difference to a classical stochastic process can be noticed in the actual observations. This was first shown for an initial vacuum (Gaussian) state [5] and then generalised to initial number eigenstates [6].

The purpose of our Letter is to provide a conceptual clarification of this question, which in our opinion should settle this issue.

The simplest example, which nevertheless contains the essential features of linear cosmological perturbations, is the case of a real massless scalar field ϕ in a $\mathcal{K} = 0$ Friedmann Universe with accelerated expansion of the scale factor a . This also comprises formally the case of metric perturbations $h_{\mu\nu}$. We use in the following the conformal time parameter η which is defined by $dt = ad\eta$, and denote derivatives with respect to η by primes. It is convenient to consider the rescaled variable $y \equiv a\phi$ (the corresponding momenta thus being related by $p_y = a^{-1}p_\phi$).

We consider first the “system” (the modes of the scalar field) without invoking any environment. The Hamiltonian can be decomposed into sums for each wave vector \mathbf{k} ,

$$\hat{H} = \int d^3\mathbf{k} (\hat{H}_{\mathbf{k}}^1 + \hat{H}_{\mathbf{k}}^2) , \quad (1)$$

where

$$\hat{H}_{\mathbf{k}}^1 = \frac{1}{2} \left(p_{\mathbf{k}1}^2 + k^2 y_{\mathbf{k}1}^2 + \frac{a'}{a} y_{\mathbf{k}1} p_{\mathbf{k}1} \right) , \quad (2)$$

and a similar expression holds for $\hat{H}_{\mathbf{k}}^2$, with the indices $\mathbf{k}1$ replaced by $\mathbf{k}2$. (We

note that $y_{\mathbf{k}1} = \text{Re}y_{\mathbf{k}}$, $y_{\mathbf{k}2} = \text{Im}y_{\mathbf{k}}$, where $y_{\mathbf{k}}$ is the Fourier transform of the field y , which is a complex variable, and similar expressions hold for the momenta.) Since we are dealing in the following with each mode separately, we denote the variable just by y (and the momentum by p).

The dynamical evolution of the quantum modes is governed by the time-dependent Schrödinger equation,

$$i\hbar\psi'(y, \eta) = \hat{H}\psi(y, \eta) . \quad (3)$$

What is the initial condition for (3)? The usual assumption is that at an “early time” η_0 (near the onset of inflation), the modes are in their adiabatic ground state. This may be an immediate consequence of quantum cosmological initial conditions [7, 8] (see below), but can also be understood as a natural assumption following, e.g., from the simple and elegant principle that the Universe was in a maximally symmetric state sometime in the past [9].

Since the initial adiabatic ground state is a Gaussian state for the wave function, this Gaussian form is preserved during the time evolution. However, due to the presence of the yp -term in (2) and the accelerated expansion of the Universe, the state becomes *highly squeezed* [10]. (Because of the two contributions in (1), this is a two-mode squeezed state, although we don’t spell this out explicitly in our notation.) The solution to (3) thus reads

$$\psi(y, \eta) = \left(\frac{\Omega_R(\eta)}{\pi} \right)^{1/4} \exp \left(-\frac{\Omega(\eta)}{2} y^2 \right) , \quad (4)$$

where $\Omega \equiv \Omega_R + i\Omega_I$, and explicit results can be given for special evolutions $a(\eta)$ [5].

Squeezed states are well known from quantum optics and can be characterised by the squeeze parameter r and the squeezing angle φ . In the cosmological case, the squeezing is

$$\Delta y = C_1 a, \quad \Delta p = \frac{|C_2|}{a} . \quad (5)$$

Here,

$$C_1 = \frac{H_k}{\sqrt{2k^3}}, \quad C_2 = -i \frac{k^{3/2}}{\sqrt{2}H_k} , \quad (6)$$

where H_k denotes the Hubble parameter evaluated at the time when the perturbation “crosses” the Hubble radius $H^{-1} = (a'/a^2)^{-1}$ during the inflationary stage. Note that in the original, physical, variables one has $\Delta\phi = C_1$ and $\Delta p_\phi = |C_2|$, but that for the relevant long wavelengths much bigger than the Hubble radius for which $k\eta \ll 1$, $C_1 \gg |C_2|\eta/a^2$. The squeezing remains very large after these large scales modes cross the Hubble radius for the second time during the matter or radiation dominated stage of the recent evolution of the Universe. Thus, in this limit, the squeezing is in the momentum (more precisely, in the quantity $\hat{p} - p_{cl}(\eta)$).

For a perturbation which will now appear on large cosmological scales, one has $r \approx 100$. This is more than one order of magnitude larger than r -values obtained in quantum optical experiments [11] (recall that r is the logarithm of the amplitude).

An important property of the state (4) is that in the limit of large squeezing one has $\Omega_I/\Omega_R \gg 1$. It follows that it then becomes a WKB state par excellence [5, 12], although extremely broad in the y -direction.

In the Heisenberg picture, the system is characterised by the presence of a “growing” and a “decaying” mode in the solutions for $\hat{y}(\eta)$ and $\hat{p}(\eta)$ [5, 12]. For large times one gets

$$\hat{y}(\eta) \rightarrow f(\eta)\hat{y}(\eta_0), \quad \hat{p}(\eta) \rightarrow g(\eta)\hat{p}(\eta_0). \quad (7)$$

Since the information about $\hat{p}(\eta_0)$ is completely lost in this limit, \hat{y} and \hat{p} approximately *commute* at late times. This, of course, is the expression of the WKB situation in the Heisenberg picture. For the modes which presently appear on large cosmological scales, the ratio of the decaying to the growing mode is $e^{-2r} \propto 10^{-100}$. In the limit (7) one also has

$$[\hat{y}(\eta_1), \hat{y}(\eta_2)] \approx 0, \quad (8)$$

i.e., $\hat{y}(\eta)$ approximately commutes at different times.

Clearly, the state (4) is a genuine quantum state and is, because of its broadness in y , very different from “classical” states such as coherent states (narrow wave packets). How, then, comes the conclusion that (4) cannot be distinguished from a classical stochastic process [5]?

The point is that observations of the cosmic microwave background anisotropies are measurements of *field amplitudes*. For the state (4), all corresponding expectation values are – in the limit of high squeezing – indistinguishable from expectation values with respect to a classical Gaussian phase space distribution [5]. One could, of course, in principle observe the quantum coherence present in (4) by relying on other observables such as $\hat{p} - p_{cl}(\eta)$. Such measurements can, however, not be performed with any of the satellite missions, and they are even very difficult to perform for similar states in the laboratory [12, 13].

The condition (8) is the condition for a so-called *quantum nondemolition measurement* (QND) [3, 14]. This means that an observable obeying (8) allows repeated measurements with great predictability. For a harmonic oscillator system such as (2), the appropriate QND observables are the “quadrature phase operators” which only in the limit of high squeezing (neglection of the decaying mode) become identical to the field amplitude. In the “Copenhagen interpretation” of quantum theory, one could express this by saying that if a measurement put the system into an eigenstate of $\hat{y}(\eta_1)$, all future measurements would give the corresponding eigenstates of $\hat{y}(\eta_2)$, $\eta_2 > \eta_1$, corresponding to the classical evolution of the system.

We note that a nice analogy to the above cosmological quantum state is provided by a system as simple as the free nonrelativistic quantum particle. Describing this by an initially narrow Gaussian packet, it becomes broader during the dynamical evolution. The interesting point is that for large times, $t \gg mx_0/p_0$, where x_0 and p_0 denote the initial expectation values of position and momentum, this Gaussian state becomes an exact WKB state, and one has $\Omega_I/\Omega_R \gg 1$. This corresponds to the limit of high squeezing in the cosmological state (4). Like there, in the limit of large times one has $[\hat{x}, \hat{p}] \approx 0$, but this time it is the information about the initial position, and not the initial momentum, that is neglected, since

$$\hat{x}(t) \rightarrow \frac{\hat{p}_0 t}{m}, \quad \hat{p}(t) = \hat{p}_0 .$$

The momentum is always a QND variable for the free particle, but for large times also position becomes such a variable, since $[\hat{x}(t_2), \hat{x}(t_1)] \approx 0$, like (8) above. Note that for a different but closely related experiment when we fix some large value of x and measure the arrival time of a particle which passed $x = x_0$ at $t = 0$, the discussed quasiclassical behaviour of \hat{x} simply means that the classical arrival time $t = (x - x_0)m/p_0$ is much bigger than its quantum indeterminacy (recently discussed in [15]) $\Delta t \sim \Delta xm/p_0 \sim \hbar/E$, where E is the particle energy.

Up to now we considered our system (the field modes) to be perfectly isolated. How realistic is such a situation? From other applications of quantum theory one knows that the effect of an environment can be very large, as far as decoherence is concerned, in situations where the dynamical influence is completely negligible [3]. A well-known example is a dust grain in intergalactic space which assumes classical behaviour already due to the weak coupling to the microwave background radiation [3, 16]. In the cosmological context, global gravitational degrees of freedom become extremely classical due to their universal coupling to all other fields [3, 17]. It is for this reason that the above framework of quantum theory on fixed backgrounds is consistent.

A quantitative measure for the entanglement of a system with its environment is the “rate of de-separation” [3]. This is defined as follows. Assume that at an initial instant ($t = 0$) the total state is a product state, $|\Psi_0\rangle = |\psi_0\rangle|\mathcal{E}_0\rangle$, where states $\{|\psi_i\rangle\}$ refer to the system – the field amplitudes in the above examples – and states $\{|\mathcal{E}_i\rangle\}$ refer to its environment (other degrees of freedom to which the system couples). The rate of de-separation, A , is defined as the coefficient in $w_0(t) = 1 - At^2$, where $w_0(t)$ is the probability to find any unentangled state. Therefore,

$$A = \sum_{j \neq 0, l \neq 0} |\langle \psi_j | \mathcal{E}_l | \hat{H} | \psi_0 | \mathcal{E}_0 \rangle|^2 , \quad (9)$$

where \hat{H} denotes here the full Hamiltonian. If $A \neq 0$, the total state is no longer a product state, and entanglement occurs.

Large entanglement with unobservable (“irrelevant”) degrees of freedom produces loss of coherence in the system. In this process, a *distinguished* basis emerges for the system (the “pointer basis”) with respect to which the system

exhibits classical behaviour. The corresponding pointer observable, $\hat{\Lambda}$, must obey

$$[\hat{\Lambda}, \hat{H}_{int}] = 0 , \quad (10)$$

where \hat{H}_{int} is the interaction Hamiltonian between system and environment. If in addition

$$[\hat{\Lambda}(t_1), \hat{\Lambda}(t_2)] = 0 , \quad (11)$$

the pointer basis is robust in time and defines a *bona fide* classical trajectory. As we have seen above, the second condition (11) is fulfilled for the field amplitudes \hat{y} because it is a QND variable, see (8). What about the first condition (10)? The answer depends of course on the interaction Hamiltonian whose precise form depends on the model under consideration. However, typical and most realistic interactions for the spacetime metric perturbations $h_{\mu\nu}$ are due to the non-linearity of the gravitational field. They are mainly proportional either to h^2 for $k\eta \ll 1$, or to $(\delta\epsilon/\epsilon)^2$ for $k\eta \gg 1$, where $\delta\epsilon/\epsilon \propto h(k\eta)^2$ is the energy density perturbation. In particular, the latter is a usual non-linearity arising in Newtonian gravity. These kinds of interactions do not depend on the field momenta. Interactions depending on the momenta also exist, e.g. scattering of gravitons $g+g \rightarrow g+g$, but their strength is exceedingly small even during the inflationary era, not to mention nowadays. Therefore, $[\hat{y}, \hat{H}_{int}] = 0$, which together with (8) leads to \hat{y} defining the classical basis.

To estimate the strength of decoherence, the de-separation parameter A has to be evaluated for the above cosmological situation. In the simplest case, the coupling of the field amplitudes \hat{y} to other degrees of freedom, denoted collectively by \hat{z} , is of the form $\hat{H}_{int} \propto g\hat{y}\hat{z}$, with g being a coupling constant. This will give a lower bound to all other, more realistic, couplings. If one, for example, starts with a state that is a product of the squeezed state (4) with a coherent state of its environment (the \hat{z} -variable), one finds

$$\begin{aligned} A &= g^2 (\cosh 2r + \cos 2\varphi \sinh 2r) \\ &\xrightarrow{r \rightarrow \infty, \varphi \rightarrow 0} g^2 e^{2r} \gg 1 . \end{aligned} \quad (12)$$

For the modes which now appear on large cosmological scales we had $e^{2r} \approx 10^{100}$, so that decoherence would only be negligible if one fine-tuned the coupling to $g^2 < 10^{-100}$, which of course is totally unrealistic.

We note that for the state (4) the de-separation parameter is proportional to the width of the Gaussian, $\Omega_R^{-1/2} = A/2kg^2$. Therefore, one recognizes again that large squeezing in p (corresponding to large broadening in y) causes a large entanglement and therefore large decoherence for the system. This is not surprising, since it is known that the most classical states in the sense of decoherence are the coherent states [3], which are very different from the high-squeezing limit. We also note that the above entanglement would only be absent for large r if one had $\varphi = \pi/2$, corresponding to an exact squeezing in the y -direction (the exact opposite case from what is happening).

The difference to the isolated case discussed above is now that even if one possessed the unrealistic capabilities of observing, for example, particle numbers instead of field amplitudes, one would never see any effect of coherences between different “classical trajectories” $|y(\eta)\rangle$ – they represent “consistent histories” [3, 18] to an excellent approximation. The quantum origin of the field modes is only reflected in the initial conditions. What is left for observations is the fundamental prediction about the Gaussian statistics of the initial metric perturbations. Note that this prediction is not restricted to vacuum initial conditions but remains valid with excellent accuracy for a much wider class of non-vacuum initial conditions [6]. It is remarkable that this prediction turns out to be in very good agreement with observations of the CMB temperature angular anisotropy [19], the spatial distribution of galaxies and their peculiar velocities [20] and the abundance of rich clusters of galaxies [21] – no definite signs of non-Gaussianity in the initial conditions for the perturbations were found.

The presence of the environment and its decohering influence gives also a perfect justification for the phenomenological use of the “Copenhagen interpretation”. However, as in usual quantum mechanics, the “Copenhagen interpretation” and the notion of a wave function collapse during a measurement are not required for any actual experiment, so this scenario can be described as well by the “many-worlds” interpretation, where decoherence guarantees the dynamical independence of the various branches.

We want to conclude with some remarks on the relevance of the above for the arrow of time [22]. The Second Law of thermodynamics only holds in the inflationary Universe because its evolution started with a state statistically very improbable but dynamically allowed – the maximally symmetric state. Quantum cosmology can, in principle, give a justification for this peculiarity. For example, taking an initial state which is unentangled with respect to the various degrees of freedom, one gets automatically, by solving the equations of quantum cosmology, a Gaussian ground state which then becomes highly squeezed during the following evolution [7, 8]. At present the Universe is still far from a maximally chaotic, or highly entropic, state. The main contribution to the total entropy S of the Universe inside the present cosmological horizon produced by space-time metric perturbations comes from gravitational waves with wavelengths of the order of 1 cm – the characteristic wavelength of CMB photons. This contribution does not exceed the entropy of CMB itself which is $S_\gamma \approx 4 \times 10^{88} h^{-3} \left(\frac{T_\gamma}{2.73 \text{ K}} \right)^3 k_B$ (each species of massless or almost massless neutrinos makes a contribution $S_\nu = \frac{7}{22} S_\gamma$ to S , the Universe is further assumed to be flat with zero cosmological constant), where $h = \frac{H_0}{100}$, while H_0 is the present Hubble constant expressed in km/s/Mpc. So, as was first noted by Penrose [23] the total entropy of the observable part of the Universe is much smaller than the entropy of a black hole with the same total mass ($\sim 10^{124} k_B$) which is believed to be the maximal possible entropy for a given mass. This large degree of order still existing in the Universe can provide a well-defined arrow of time.

To summarise, although the observed fluctuations in the cosmic microwave background radiation are of quantum origin, for us they appear to be of a perfectly classical nature, and it is impossible to observe any quantum coherences between them. The above treatment shows the universal nature of quantum concepts – the physics of the early Universe is not more exotic than the physics of quantum optics.

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